The Mathematical Association

Primary Mathematics Challenge

Bonus Paper

Answers and Notes February 2013

These notes provide a brief look at how the problems can be solved. There are sometimes many ways of approaching problems - not all can be given here. Suggestions for further work based on some of these problems are also provided.

P1	A	0.00001	This is the longest number but the smallest in value.
P2	C	15	30 sec is one-tenth of 5 min so one-tenth of the heads will be stitched on.
1	**B**	£1.60	Two 7½ minutes make up 15 minutes, so in one hour I have to pay 8 times 20p = £1.60.
2	**E**	12	Cutting off a small triangle adds one extra side to each of the six sides of the hexagon. So after cutting off the six small triangles there will be 12 sides.
3	**A**	55	Pupils could add up the ten numbers directly. Or add the first and the last to make 11, the second and the second to last to make another 11 and so on, getting five groups totalling 11 each. The sum of the ten numbers is therefore 5 × 11 = 55.
4	**E**	120	Four circuits of the track make 1km so 120 circuits make 30km.
5	**C**	$40cm^2$	Adding the diagonals to the smaller square shows that the area of the larger square is twice the area of the smaller square.
6	**A**	1/8	Mr Oak uses 'solid' 1 - (½ + ¼ + ⅛) = ⅛ of the time. Pupils could add the fractions in the bracket using equivalent fractions, then subtract from 1.
7	**A**	Monday	June has 30 days (4 × 7 + 2) so the day of the week moves on two days between June and July.
8	**C**	1800	Pupils might calculate the volumes of the two suitcases (18 $000cm^3$ and 54 $000cm^3$) and see that the volume of the larger is three times to volume of the smaller suitcase. So the number of sprouts will be 3 × 600 = 1800. But pupils might see that, in effect, one side has been doubled (to make 1200 sprouts) and

			then multiplied by 1.5 to make 1800 sprouts.
9	**B**	13	The two-digit primes less than 20 are 11, 13, 17 and 19. Reversing the digits gives 11, 31, 71 and 91. The only pair to give 403 when multiplied is 13 and 31.
10	**A**	Aled	Aled is stretched by 5cm. Bertie, Daisy and Ethan are stretched by only 3cm, so rule them out. That leaves just Aled and Carmen. Carmen is taller than Aled and so her stretch is proportionally smaller.
11	**C**	99	There are palindromic numbers before the year 3000, so let us start with a 2. The number must end with a two, and the two middle numbers must be as low as possible and the same. The year 2002 has passed, but 2112 has not. 2112 – 2013 = 99 years.
12	**A**	4.5m^2	If the diagram is increased to make two squares, the difference in areas is 9m^2. We need half that difference giving 4.5m^2.
13	**D**	20	In a 24 day period, one foot needs 12 socks and the other needs 8 socks. The total is 20 socks.
14	**B**	2	The second to last digit is always 2. For example 5^3 is 125. When multiplying this by 5, we see that is it inevitable that the second to last digit remains 2. Try repeated multiplication of 5 on a calculator.
15	**D**	12	The minute hand takes one hour per revolution; the hour hand takes twelve hours. So the minute hand goes round 12 times faster than the hour hand.
16	**D**	1	Adding the first three equations gives $a + b + c + d + e + f = 6$. Comparison with the fourth equation gives $a = 1$.
17	**E**	4199	The final digit of the answer will be the same as the final digit to $3 \times 7 \times 9$ which is 9. So the four-digit number is 4199.
18	**B**	89p	It isn’t necessary to calculate the price of each cake and doughnut. Subtracting the two ‘sentences’ tells us that three cakes and one doughnut will cost £2.06 – £1.17 = 89p. In fact, cakes cost 25p each and doughnuts cost 14p each.
19	**D**	68	After the first three numbers, each number is the sum of the previous three numbers. So the next number is 11 + 20 + 37 = 68.

20 **E** 8Pupils might start by choosing values for x and calculate how long Mr Urly worked. It turns out that whatever value is chosen for x, the answer is always 8.
By algebra, we calculate that, in the morning he worked $12 - (x + 2) = 10 - x$ hours. In the afternoon he worked $x - 2$ hours. The total is $(10 - x) + (x - 2)$ and the *x*s cancel out to give 8 hours.

21 **B** 18 Working from the point on the ground on the left, every 4m along gives 2m in height. The distance along the ground is 36m, so the height of the tower is 9 × 2 = 18m.
Alternatively, by ratio, $h/36 = 2/4$ giving $h = 18$.

22 **C** 1 : 4 : 15 Subtracting living costs gives the figures for other expenses as £4000, £16 000 and £60 000. These are in a ratio of 4 : 16 : 60. That is 1 : 4 : 15 .

23 **D** 2160 In 30 hours the snail travels one mile. In this time the cheetah travels 72 × 30 = 2160 miles. So the cheetah is travelling 2160 times faster than the snail.

24 **A** 26 *a* must be 9 giving 7 × 6 = 3 'carry' 6.
Multiplying 7 by 6 adding the carried 6 gives 48 so $c = 4$.
Write down the 0. 8 × 9 is 72 so $b = 8$.
This gives $d = 5$. The total of the four numbers is 9 + 8 + 4 + 5 = 26.

```
  6 a ×
    b 7
  c 8 3
d 5 2 0
6 0 0 3
```

25 **B** £10 There are three winning scores greater than ten: 11, 11 and 12 (draw the 6 by 6 grid of outcomes when two die are thrown).
So, in 36 games, the player pays out £36 and receives £30, losing £6 on average. In 12 games she would therefore expect to lose £2. And in 60 games would therefore expect to lose £10.

Some possibilities for further problems

2 At the end of Q2, the polygon has 12 edges. If the process was repeated, there would be 24 edges. And if this carried indefinitely, how many edges would there be and approximately what would the shape of the polygon now look like?

3 Here are some similar problems:

a) 10 + 11 + 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19
b) 23 + 24 + 25 + 26 + 27 + 28 + 29 + 30 + 31 + 32
c) 10 + 12 + 14 + 16 + 18 + 20 + 22 + 24 + 26 + 28
d) 35 + 38 + 41 + 44 + 47 + 50 + 53 + 56 + 59 + 62

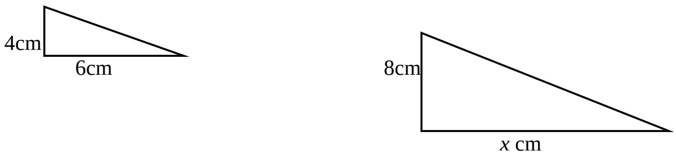
6 The fractions in this problem provide an easy insight into the situation in which the sum of an infinite number of numbers is finite. In this case, adding more of these fractions will only get closer to the number 2.
Pupils might consider if the sum can actually reach 2; if the sum did continue for ever, would it reach 2?
There are other sums which, if continued for ever, would reach a finite number: 1/3 + 1/9 + 1/27 + 1/81 + + + .
Pupils might instigate what happens in this case: 1 + 1/2 + 1/3 + 1/4 + 1/5 ... .

14 Pupils can investigate what happens when other numbers are raised to different powers. What rules can they discover with the first digits, second digits etc?

17 In Q17, three numbers are used. Suppose the code used only <u>two</u> prime numbers. Can pupils find them for these numbers? 143 (11 x 13), 221 (13 x 17), 2773 (47 x 59). It does get difficult very quickly!

18 Here are more simultaneous equations. One pair is impossible to solve!
a) $4c + 3d = 24$ and $3c + 3d = 21$ b) $6c + 5d = 60$ and $3c + 2d = 27$
c) $6c + 4d = 38$ and $3c + 2d = 19$

21 Here are two easy similar triangles. How long is the side marked *x*?



22 The gap between top and bottom earners in our country is still growing. Suppose the three people in this question were all given a 5% pay rise. How much increase in

pay

would they get? Can your pupils find a way of giving pay increases which *they* think is fair?

25 Expectation is the topic here. It is the expected loss and may not be the actual loss. Some games have an expectation of zero – break even. Betting on heads of tails for

a

balanced die has a zero expectation. Can your pupils design any other games with zero expectation?
Some games such as slot machines have an expectation fixed by law. Others, such as roulette are fixed by their design. In these games there are many losers but some people manage to win.
The national lottery has a negative expectation, and most people lose.
Why do some people think gambling is bad?
Expectation is a mathematical idea which has uses apart from gambling, and is an important topic in probability.